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Neutrino Oscillations in Caianiello's Quantum Geometry Model

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Neutrino flavor oscillations are analyzed in the framework of Quantum Geometry model proposed by Caianiello. In particular, we analyze the consequences of the model for accelerated neutrino particles that experience an effective Schwarzschild geometry modified by the existence of an upper limit on the acceleration, which implies a violation of the equivalence principle. We find a shift of quantum-mechanical phase of neutrino oscillations, which depends on the energy of neutrinos as E^3. Implications on atmospheric and solar neutrinos are discussed.

1. INTRODUCTION

The long-standing problem about the deficiency of the solar neutrino and the atmospheric neutrino might be explained invoking oscillations between the various flavors or generations of neutrinos. In fact, neutrino oscillations can occur in vacuum if the eigenvalues of the mass matrix are not all degenerate, and the corresponding mass eigenstates are different from weak interaction eigenstates (Bilenky and Pontecorvo, 1978). The most discussed version of this type of solutions is the MSW effect (Mikhyev and Smirnov, 1986a,b; Wolfenstein, 1978), in which solar electron neutrinos are converted almost completely into muon or tau neutrinos, owing to the presence of matter in the Sun.

Recently, quantum-mechanical oscillations of neutrinos propagating in a gravitational field (usually the Schwarzschild field) has been discussed by several authors (e.g., Ahluwalia and Burgad, 1996; Bhattacharya *et al.*, 1999, and

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references therein) in view of astrophysical consequences also. Ahluwalia and Burgard consider, in fact, the gravitational effect on oscillations, showing that an external weak gravitational field of a star adds a new contribution to the phase difference (Ahluwalia and Burgard, 1996). They also suggest that the new oscillation phase may be a significant effect on the supernova explosions because the extremely large fluxes of neutrinos are produced with different energies corresponding to the flavor states. This result has been also discussed by Bhattacharya *et al.* (1999). In their approach it is shown that the possible gravitational effect appears at the higher order with respect to one calculated in Ahluwalia and Burgard (1996), with a magnitude of the order 10^{-9}, which is negligible in typical astrophysical applications.

An alternative mechanism of neutrino oscillations has been proposed in Gasperini (1988, 1989) and Halprin and Leung (1991) as a means to test the equivalence principle. In this mechanism, neutrino oscillations follow by assuming a flavor nondiagonal coupling of neutrinos to gravity, which violates the equivalence principle, that is, if the universality of the gravitational couplings to different flavors breaks down, additional phase difference appears. Therefore, understanding how presence of a gravitational field or violation of the equivalence principle affects neutrino oscillation phase is an important matter.

In this paper we face this issue in the framework of Quantum Geometry model proposed by Caiainiello some years ago in an attempt to unify Quantum Mechanics and General Relativity principles (see Caianiello, 1981, 1992, and references therein). In this model the effective four-dimensional metric depends on the mass of a given test particle, so that *test particles with different rest masses experience different geometries and, as consequence, an effective violation of the equivalence principle occurs*. The geodesic paths along which test particles are moving become mass-dependent, resulting in a nonuniversality of the gravitational coupling (Caianiello *et al.*, 1990a), and making the metric observer-dependent, as also conjectured by Gibbons and Hawking (1977).

The view frequently held that proper acceleration of a particle is limited upwardly (Caianiello *et al.*, 1982) finds in this model a geometrical interpretation epitomized by the line element

$$d\tilde{s}^2 = \left(1 + \frac{g_{\mu\nu} d\ddot{x}^\mu d\ddot{x}^\nu}{\mathcal{A}_m^2}\right) ds^2 \equiv \sigma^2(x)\, ds^2, \tag{1}$$

experienced by the accelerating particle along its worldline. In (1) $\mathcal{A}_m = 2mc^3/\hbar$ is the proper Maximal Acceleration (MA) of the particle of mass m and four-acceleration $\ddot{x}^\mu$.

MA has several implications. It provides a regularization method in Quantum Field Theory (Feoli *et al.*, 1999b), allowing to circumvent inconsistencies associated with the application of pointlike concept to relativistic quantum particles, it is the same cutoff on the acceleration required in an ad hoc fashion by Sanchez

(1993) to regularize the entropy and free energy of quantum strings, and it is also invoked as a necessary cutoff by McGuigan in the calculation of black hole entropy (McGuigan, 1994).

Applications of Caianiello's model include cosmology (Caianiello *et al.*, 1991; Capozziello *et al.*, 1999), where initial singularity can be avoided while preserving inflation, dynamics of accelerated strings (Feoli, 1993), and energy spectrum of a uniformly accelerated particle (Caianiello *et al.*, 1990b).

The extremely large value that $\mathcal{A}_m$ takes for all known particles makes a direct test of the model difficult. Nonetheless, a direct test that use photons in a cavity has also been suggested (Papini *et al.*, 1995). More recently, we have worked out the consequences of the model for classical electrodynamics of a particle (Feoli *et al.*, 1997), mass of the Higgs boson (Kuwata, 1996; Lambiase *et al.*, 1999), and Lamb shift in hydrogenic atoms (Lambiase *et al.*, 1998). In the last instance, agreement between experimental data and MA corrections is very good for H and D. For He^+ agreement between theory and experiment is improved by 50% when MA corrections are included. MA effects in muonic atoms appear to be measurable in planned experiments (Chen *et al.*, 1999). MA also affects the helicity and chirality of particles (Chen *et al.*, 2000). Very recently the behavior of classical (Feoli *et al.*, 1999a) and quantum (Capozziello *et al.*, 2000a) particles in a Schwarzschild field with MA modifications has been studied.

A limit on acceleration also occurs in string theory. Here the upper limit manifests itself through Jeans-like instabilities (Gasperini *et al.*, 1991; Sanchez and Veneziano, 1990) that occur when acceleration induced by the background gravitational field is larger than a critical value $a_\mathrm{c} = (m\alpha)^{-1}$ for which string extremities become causally disconnected (Gasperini, 1992). String mass is m and string tension α. Frolov and Sanchez (Frolov and Sanchez, 1991) have then found that a universal critical acceleration a_c must be a general property of strings. It is worth to note that it is possible to derive, in the framework of Caianiello's Quantum Geometry model, the generalized uncertainty principle of string theory (Capozziello *et al.*, 2000b).

The paper is organized as follows. In Section 2 we shortly discuss the Quantum Geometry model and derive the modified Schwarzschild geometry by taking into account the MA corrections (for details see Feoli *et al.*, 1999a). In Section 3 we calculate the corrections induced by MA to the quantum-mechanical phase of mixed states of neutrinos radially propagating in the modified Schwarzschild geometry. Conclusions are drawn in Section 4.

2. MODIFIED SCHWARZSCHILD SPACE–TIME IN QUANTUM GEOMETRY

The model proposed by Caianiello, which includes the effects of MA in dynamics of particles, was to enlarge the space–time manifold to an eight-dimensional space–time tangent bundle TM_8. In this way the invariant line element is defined as (Caianiello *et al.*, 1990a)

$$d\tilde{s}^2 = g_{AB}\, dX^A\, dX^B, \quad A, B = 1, \ldots, 8, \tag{2}$$

where the coordinates of TM_8 are

$$X^A = \left(x^\mu ; \frac{1}{\mathcal{A}_m} \frac{dx^\mu}{ds} \right), \quad \mu = 1, \ldots, 4, \tag{3}$$

and

$$g_{AB} = (g_{\mu\nu};\, g_{\mu\nu}), \qquad ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu. \tag{4}$$

ds is the ordinary line element of four-dimensional space–time and dx^μ/ds is the four-velocity of the particle moving along its worldline. In Eq. (3), $\mathcal{A}_m$ is the MA depending, in the quantum geometry theory proposed by Caianiello, on mass m of the particle, whose value is given by $\mathcal{A}_m = 2mc^3/\hbar$. In other models, $\mathcal{A}_m$ is interpreted as a universal constant and m is replaced by Plank mass m_{P}. Using Eqs. (3) and (4) the line element (2) can be written as

$$d\tilde{s}^2 = \left(1 + \frac{g_{\mu\nu} \ddot{x}^\mu \ddot{x}^\nu}{\mathcal{A}_m^2} \right) g_{\alpha\beta}\, dx^\alpha\, dx^\beta \equiv \sigma^2(x) g_{\alpha\beta}\, dx^\alpha\, dx^\beta, \tag{5}$$

where $\ddot{x}^\mu = d^2x^\mu/ds^2$ is the four-acceleration of particles and $ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu$ is the metric due to a background gravitational field. In absence of gravity, $g_{\mu\nu}$ is replaced by Minkowski metric tensor $\eta_{\mu\nu}$. The embedding procedure has been developed to find the effective space–time geometry in which a particle can move when the constraint of an MA is present (Caianiello *et al.*, 1990b). In fact, if one finds the parametric equations that relate velocity field $\dot{x}^\mu$ to the first four coordinates x^μ, one can calculate the effective four-dimensional metric on the hypersurface locally embedded in TM_8. This procedure strongly depends on the choice of velocity field of the particle. From Eq. (5) it follows also that even starting from a phase space TM_8 with a flat metric, that is, $g_{AB} = (\eta_{\mu\nu};\, \eta_{\mu\nu})$, in case of accelerating particles characterized by a velocity field $\dot{x}^\mu$ not trivially constant, one gets an effective four-dimensional geometry that, in general, is curved. In other words, even though background space–time is flat, the effective geometry experienced by an accelerating particle is curved.

We stress that the curvature of effective geometry is not induced by matter through conventional Einstein equations: It is because of the motion in momentum space and vanishes in the limit $\hbar \to 0$. Thus, it represents a quantum correction to given background geometry.

To calculate the corrections to the Schwarzschild field experienced by a particle initially at infinity and falling toward the origin along a geodesic, one must calculate the metric induced by the embedding procedure (5). On choosing $\theta = \pi/2$, one finds the conformal factor produced by the embedding procedure

$$\sigma^2(r) = 1 + \frac{1}{\mathcal{A}_m^2}\left[\left(1 - \frac{2M}{r}\right)\ddot{t}^2 - \frac{\ddot{r}^2}{1 - 2M/r} - r^2\ddot{\phi}^2\right], \tag{6}$$

where $\ddot{t}$, $\ddot{r}$, and $\ddot{\phi}$ are given by the standard results (Misner *et al.*, 1973)

$$\begin{aligned}
\ddot{t}^2 &= \frac{\tilde{E}^2}{(1 - 2M/r)^4}\frac{4M^2}{r^4}\left[\tilde{E}^2 - \left(1 - \frac{2M}{r}\right)\left(1 + \frac{\tilde{L}^2}{r^2}\right)\right],\\
\ddot{r}^2 &= \left(-\frac{M}{r^2} + \frac{\tilde{L}^2}{r^3} - \frac{3M\tilde{L}^2}{r^4}\right)^2,\\
\ddot{\phi}^2 &= \frac{4\tilde{L}^2}{r^6}\left[\tilde{E}^2 - \left(1 - \frac{2M}{r}\right)\left(1 + \frac{\tilde{L}^2}{r^2}\right)\right].
\end{aligned} \tag{7}$$

M is mass of the source, $\tilde{E}$ and $\tilde{L}$ are total energy (E) and angular momentum (L) per unit of particle mass m, respectively. The conformal factor $\sigma^2(r)$ is then given by (Feoli *et al.*, 1999a)

$$\begin{aligned}
\sigma^2(r) = 1 + \frac{1}{\mathcal{A}_m^2}\Bigg\{&-\frac{1}{1 - 2M/r}\left(-\frac{3M\tilde{L}^2}{r^4} + \frac{\tilde{L}^2}{r^3} - \frac{M}{r^2}\right)^2\\
&+ \left(-\frac{4\tilde{L}^2}{r^4} + \frac{4\tilde{E}^2M^2}{r^4(1 - 2M/r)^3}\right)\left[\tilde{E}^2 - \left(1 - \frac{2M}{r}\right)\left(1 + \frac{\tilde{L}^2}{r^2}\right)\right]\Bigg\}.
\end{aligned} \tag{8}$$

Modifications to Schwarzschild geometry experienced by radially ($\tilde{L} = 0$ accelerating neutrinos are easily calculated. In fact, from Eq. (8) and by using weak field approximation, one gets

$$\sigma^2(r) = 1 - \frac{1}{\mathcal{A}_m^2}\left(\frac{1}{4} + \frac{E^2}{m^2} - \frac{E^4}{m^4}\right)\frac{r_s^2}{r^4}, \tag{9}$$

where $r_s = 2GM/c^2$ is Schwarzschild radius.

3. MA CORRECTIONS TO QUANTUM-MECHANICAL PHASE

Corrections induced by MA to the quantum-mechanical phase mixing of massive neutrinos are calculated following Bhattacharya *et al.* (1999). In the semiclassical approximation, where the action of a particle is considered as a quantum phase, a particle propagating in a gravitational field from a point A to a point B changes its quantum mechanical phase according to the relation (Stodolski, 1979)

$$\Phi = \frac{1}{\hbar}\int_A^B m\; d\tilde{s} = \frac{1}{\hbar}\int_A^B p_\mu\; dx^\mu. \tag{10}$$

Here $p_\mu = m\tilde{g}_{\mu\nu}(dx^\nu/d\tilde{s})$ is four-momentum of the particle and $\tilde{g}_{\mu\nu} = \sigma^2(r)g_{\mu\nu}$, where the conformal factor $\sigma^2(x)$ is defined in Eq. (9). In order that different neutrinos could interfere at the same final point B, with coordinates (t_B, r_B), one requires, in the geometrical optical approximation, that relevant components of the wave function have not started from the same initial point A, with coordinates (t_A, r_A). Then, the quantum-mechanical phase becomes

$$\Phi = \frac{1}{\hbar}\int_{r_A}^{r_B} p_r\ dr. \tag{11}$$

Inserting momentum of the particle, calculated by mass-shell condition $\tilde{g}^{\mu\nu}p_\mu p_\nu = m^2$,

$$p_r = \frac{\sqrt{E^2 - m^2\sigma^2(1 - r_\mathrm{s}/r)}}{1 - r_\mathrm{s}/r} \tag{12}$$

into Eq. (11) one gets, up to second order in r_s/r,

$$\Phi = \Phi_0 + \Phi_{\mathcal{A}_m} \tag{13}$$

where

$$\begin{aligned}\Phi_0 = {} & \frac{\sqrt{E^2 - m^2}}{\hbar}(r_B - r_A) + \frac{(2E^2 - m^2)r_\mathrm{s}}{2\sqrt{E^2 - m^2}}\ \log\frac{r_\mathrm{s}}{r}\\ & - \frac{r_\mathrm{s}^2\sqrt{E^2 - m^2}}{\hbar}\left(1 + \frac{m^2}{2(E^2 - m^2)} + \frac{m^4}{8(E^2 - m^2)^2}\right)\left(\frac{1}{r_B} - \frac{1}{r_A}\right),\end{aligned} \tag{14}$$

represents the result of Bhattacharya *et al.* (1999), and

$$\Phi_{\mathcal{A}_m} = \frac{1}{\mathcal{A}_m^2}\left(\frac{1}{4} + \frac{E^2}{m^2} - \frac{E^4}{m^4}\right)\frac{m^2 r_\mathrm{s}^2}{6\hbar\sqrt{E^2 - m^2}}\left(\frac{1}{r_B^3} - \frac{1}{r_A^3}\right) \tag{15}$$

is the contribution due to the MA. For ultrarelativistic neutrinos, $E \gg m$, the relative quantum-mechanical phase $\Delta\Phi$ of the two different mass eigenstates is given by

$$\Delta\Phi = \Delta\Phi_{(0)} + \Delta\Phi_{\mathcal{A}_m}, \tag{16}$$

where

$$\Delta\Phi_{(0)} = \frac{\Delta m^2}{2E\hbar}(r_B - r_A) + \frac{\Delta m^2}{4E^2}(r_B - r_A) - \frac{\Delta m^2\left(m_1^2 + m_2^2\right)r_\mathrm{s}}{8\ \hbar E^3}\ \ \log\frac{r_B}{r_A}, \tag{17}$$

as in Bhattacharya *et al.* (1999), and

$$\Delta\Phi_{\mathcal{A}_m} = \frac{\hbar E^3}{24}\frac{\Delta m^2\left(m_1^2 + m_2^2\right)}{m_1^4 m_2^4}\frac{r_\mathrm{s}^2\left(r_B^3 - r_A^3\right)}{r_B^3 r_A^3}. \tag{18}$$

Here $\Delta m^2 = |m_2^2 - m_1^2|$. In Eq. (17), the first term represents the standard phase of neutrino oscillations, the second term is the kinetic correction to the first order, and finally, the last term is the gravitational correction to the leading order. The second and third term in Eq. (17) can be neglected with respect to the first term, so that we will neglect them in what follows. Notice that $\Delta\Phi_{\mathcal{A}_m} \to 0$ as $\hbar \to 0$. It is more convenient to rewrite phases (17) and (18) in the following way

$$\Delta\Phi_{(0)} = 2.5 \times 10^3 \frac{\Delta m^2}{\text{eV}^2} \frac{\text{MeV}}{E} \frac{r_A - r_B}{\text{km}}, \tag{19}$$

and

$$\Delta\Phi_{\mathcal{A}_m} = 2.4 \times 10^8 \frac{\Delta m^2}{\text{eV}^2} \frac{E^3}{\text{MeV}^3} \frac{M^2}{M_\odot^2} \frac{\text{eV}^6}{(m_1 m_2)^4/\left(m_1^2 + m_2^2\right)} \times \frac{\text{km}^3}{(r_A r_B)^3/\left(r_A^3 - r_B^3\right)}, \tag{20}$$

where $M_\odot$ is the solar mass.

Comparison between quantum-mechanical phases (19) and (20) for atmospheric neutrinos with mass-squared difference $\Delta m^2 = (10^{-2} \div 10^{-3})$ eV^2 are reported in Table I. We have assumed the following numerical values: $r_A = R_{\text{Earth}} = 6.3 \times 10^3$ km and $r_B = r_A + 10$ km, $r_s \sim 10^{-6}$ km is the Schwarzschild radius for Earth and, finally, the energy of neutrinos is $E \sim 1$ GeV. MA corrections to the quantum-mechanical phase are meaningful for neutrinos with masses $m_1, m_2 \sim 0.05 \div 0.1$ eV. In this range, in fact, such corrections turn out to be $10^{-2} \div 10^{-3}$ smaller than the phase (17).

For solar neutrinos, we have a similar situation. Results are summarized in Table II for the values $\Delta m^2 = (10^{-10} \div 10^{-12})$ eV^2, $r_A = R_{\text{Earth}}$, $r_B = r_A + 1{,}5 \times 10^8$ km, $E \sim 1$ MeV and $E \sim 10$ MeV, $M \sim M_\odot$. Again, the quantum

Table I. Quantum-Mechanical Phase Mixing for Atmospheric Neutrinos with Fixed Value of Δm^2 and $E \sim 1$GeV

m_1	m_2	Δm^2	$\Delta\Phi_{(0)}$	$\Delta\Phi_{\mathcal{A}_m}$
0.5	0.51	10^{-2}	$2.5 \cdot 10^{-1}$	10^{-8}
0.1	0.14	10^{-2}	$2.5 \cdot 10^{-1}$	$8.6 \cdot 10^{-5}$
0.05	0.11	10^{-2}	$2.5 \cdot 10^{-1}$	$1.8 \cdot 10^{-3}$
0.01	0.1	10^{-2}	$2.5 \cdot 10^{-1}$	1.14
0.5	0.501	10^{-3}	$2.5 \cdot 10^{-2}$	10^{-9}
0.1	0.104	10^{-3}	$2.5 \cdot 10^{-2}$	$2 \cdot 10^{-5}$
0.05	0.06	10^{-3}	$2.5 \cdot 10^{-2}$	$8.9 \cdot 10^{-4}$
0.01	0.03	10^{-3}	$2.5 \cdot 10^{-2}$	1.13

Note. m_1 and m_2 are expressed in eV.

Table II. Quantum-Mechanical Phase Mixing for Solar Neutrinos

m	E	$\Delta\Phi_{(0)}/\Delta m^2$	$\Delta\Phi_{\mathcal{A}_m}/\Delta m^2$
0.5	1	$2.5 \cdot 10^{11}$	10^2
0.1	1	$2.5 \cdot 10^{11}$	$2 \cdot 10^6$
0.05	1	$2.5 \cdot 10^{11}$	$1.2 \cdot 10^8$
0.01	1	$2.5 \cdot 10^{11}$	$2 \cdot 10^{12}$
0.5	10	$2.5 \cdot 10^{10}$	10^5
0.1	10	$2.5 \cdot 10^{10}$	$2 \cdot 10^9$
0.05	10	$2.5 \cdot 10^{10}$	$1.2 \cdot 10^{11}$
0.01	10	$2.5 \cdot 10^{10}$	$2 \cdot 10^{15}$

Note. Here $m_1 \sim m_2 \sim m$ are expressed in eV and E in MeV.

mechanical phase corrections induced by MA become relevant for neutrino masses of the order $0.05 \div 0.1$ eV.

Masses below 0.05 eV lead to high corrections that cannot be treated in this perturbative model.

It is worthwhile to point out the different dependence on the energy of the two phases: $\Delta\Phi_{(0)} \sim E^{-1}$ and $\Delta\Phi_{\mathcal{A}_m} \sim E^3$. This can notably help the separation of the two components in experimental tests, because the weight of MA corrections is largely affected by the energy of neutrinos. A good statistical analysis could succeed in bringing this term to light.

4. CONCLUSIONS

Einstein's equivalence principle plays a fundamental role in the construction and testing of theories of gravity. Though verified experimentally to better than a part in 10^{11} for bodies of macroscopic dimensions, doubts have at times been expressed as to its validity down to microscopic scales. It is conceivable, for instance, that the equality of inertial and gravitational-mass break down for antimatter, or in quantum field theory at finite temperatures (Donoghue *et al.*, 1984, 1985). Einstein's equivalence principle is also violated in the Quantum Geometry model developed by Caianiello as a first step toward the unification of Quantum Mechanics and General Relativity. The model interprets quantization as curvature of the eight-dimensional space–time tangent bundle TM_8. In this space the standard operators of the Heisenberg algebra are represented as covariant derivatives and the quantum commutation relations are interpreted as components of the curvature tensor.

In this paper we have analyzed the oscillation phenomena of neutrinos propagating in a Schwarzschild geometry modified by the existence of MA, which implies a violation of the equivalence principle. We have calculated the quantum-mechanical phase showing that, for the consistence of the Caianiello model, our results are compatible with estimations of the neutrino masses giving $m_\nu \sim 0.05 \div 1$ eV.

Eqs. (19) and (20) allow to calculate the flavor oscillation probability, which is given by

$$P_{\nu_e \to \nu_\mu} = \sin^2 2\theta \, \sin^2 \left(\pi \frac{\Delta r}{\lambda_{\mathcal{A}_m}} \right), \tag{21}$$

where θ is the mixing angle, and $\lambda_{\mathcal{A}_m}$ is the oscillation length defined as (for simplicity we use the natural units $\hbar = c = 1$)

$$\lambda_{\mathcal{A}_m}^{-1} = \frac{\Delta m^2}{4E\pi} + \frac{E^3}{24\pi} \frac{\Delta m^2 \left(m_1^2 + m_2^2\right)}{m_1^4 m_2^4} \frac{r_s^2 \left(r_A^2 + r_A r_B + r_B^2\right)}{r_A^3 r_B^3}. \tag{22}$$

As well known, in the cases of interest, the oscillation length λ does depend on the energy of neutrinos as $\lambda^{-1} \sim E^n$ (Fogli *et al.*, 1999). Then $\lambda_{\mathcal{A}_m}^{-1}$ corresponds to standard oscillation plus the equivalence principle violation induced by the existence of MA, $(n = -1) \oplus (n = 3)$. The behaviour $\lambda^{-1} \sim E^{-1}$ coming from a flavor depending on the coupling to the gravitational field, as proposed by Gasperini (1988, 1989) and Halprin and Leung (1991), appeared to fit the SuperKamiokande data, as well as the other alternative mechanisms (Barger *et al.*, 1999; Choubey and Goswami, 2000; Foot *et al.*, 1998). Nevertheless, a different analysis of such data, including, for example, upward-going muons events, has been performed in Fogli *et al.* (1999) and Lipari and Lusignolo (1999). In these papers, it is shown that the best fit does confirm, at least for atmospheric neutrinos, the standard scenario as the dominant oscillation mechanism, whereas the equivalence principle violation, as formulated in Gasperini (1988, 1989) and Halprin and Leung (1991), do not provide a viable description of data.

Unlike the mechanism proposed in Gasperini (1988, 1989) and Halprin and Leung (1991), in this paper we have suggested an alternative mechanism for introducing, in the framework of Quantum Geometry, a violation of the equivalence principle in the neutrino oscillation physics. The main consequence of this approach, as shown in Eq. (22), is a different behaviour of the inverse of the oscillation length as a function of the energy ($\sim E^3$) with respect to the one obtained in Gasperini (1988, 1989) and Halprin and Leung (1991), whose energy dependence has the functional form $(E \Delta f \phi)^{-1}$, where ϕ is the constant gravitational field and Δf the measure of the violation of the equivalence principle.

Even though many efforts have been made till now for solving the neutrino oscillation problem, a definitive solution is far from achieved. Much more studies are necessary for understanding the origin of neutrino masses, the mixing of states, and the analysis of collaborations involving in neutrino oscillations experiments. Only the future generation of experiments could provide new data for probing the E^3-dependence induced by MA corrections, allowing to establish whether the violation of the equivalence principle discussed in this paper occurs, and as a consequence, whether the Quantum Geometry model proposed by Caianiello is a concrete step towards an unified theory of Quantum Mechanics and General Relativity.

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